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13. ABSTRACT

A mathematical model for triadic conflict was developed. The model was based on the offensive and defensive capabilities possessed by each of the three participants. Included in these capabilities were probability of a successful attack, damage done by a successful attack, probability of intercepting an attack, and ability to withstand an attack. The model was then tested on data collected in three experimental situations employing the triadic conflict game, the truel.

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A MODEL FOR RELATIVE CONFLICT

by

James L. Phillips, Steven G. Cole, and E. Alan Hartman

Report 72-2

Cooperation/Conflict Research Group
Michigan State University

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Conflict, whether it be between persons, small groups, organizations, or nation-states, can be understood to exist on a variety of levels. Perhaps the most extreme or intense level of conflict exists when the parties to the conflict see their own goals as absolutely incompatible with the goals of their opponents. Such a level of conflict can be described in terms of an indivisible payoff structure (Phillips and Nitz, 1968; Nitz and Phillips, 1969). An indivisible payoff structure differs from a zero-sum or even a constant-sum situation. It implies a situation in which at most one party can achieve its objective and in which all parties may fail to do so. The spectre of two nuclear powers involved in an altercation in which one seeks to establish a world-wide "workers' revolution" while the other sets about to "make the world safe for Democracy" is an example of this intense level of conflict of colossal, and terrifying, proportions. A more mundane example would be the two 19th century nobelmen who set out on a chilly morning each to "satisfy his honor" with a dueling pistol. It is, in fact, the duel that provides the prototype of conflict with indivisible payoffs.

In turning our attention to the study of such a situation, as a distinct form of conflict, we do not mean to suggest that there is anything inevitable about it. We do suggest, however, that in a period of escalating conflict there always exists the possibility that goals will be redefined in such a way as to produce this indivisibility. We further suggest that, faced with an apparently indivisible payoff structure, it is usually possible to find alternatives that mediate the conflict. The analysis of this latter process, however, is a problem of negotiation and bargaining which we propose to defer in this paper. Thus, the scope of the present undertaking will be restricted to an analysis of the dynamics of conflict under an indivisible payoff structure.

Given this restriction the analysis of the two-person situation--the duel--is rather trivial. The two participants simply exchange attacks until either (1) one wins and the other loses, (2) both are unable to continue (i.e., both lose), or (3) the conditions of conflict are mediated and the situation changes. Less trivial is the analysis of the n-person situation ($n > 2$), or n-uel.

The present paper focuses upon what we have called <u>uelative conflict</u>. This term refers to the type of conflict present in an n-uel. We will first discuss the laboratory simulation of uelative conflict, and then develop a mathematical model for strategy selection under uelative conditions. This model is based on factors inherent in the structure of power relations and on psychological assumptions about the dynamics of interaction under these conditions. The model will then be restricted to several special cases and these compared with empirical data.

Laboratory Simulation of Uelative Conflict

Although it is doubtful if pure uelative conflict has ever existed between nation states, it can be argued that certain states of affairs (e.g., war) bear a strong resemblance to it. Moreover, potential uelative conflict may very well exist at all times. The problem of how a potential uelative situation moves to a purely uelative conflict is one that we hope to deal with in subsequent work. At present, however, we restrict ourselves to the pure case. Since "real-world" examples of pure uelative conflict are infrequent and do not readily lend themselves to analysis, we have turned to the psychological laboratory where such conflicts can be created or simulated.

There has been a modest effort in recent years to develop laboratory simulations of uelative conflict. The studies by Cole (1968), Cole and Phillips (1967) and Willis and Long (1967) provide a paradigm for the study of the three person form of uelative conflict--a truel. This paradigm employs three persons as subjects or participants in a simple game which can easily be generalized to an n-person game. In this game, each participant is assigned a number of markers or points, as well as a certain capability for destroying the markers of his opponents. Each player has some defensive capability. These various capabilities are designated the resources of each player. The object of the game is for each player to retain some of his markers after his opponents markers have been completely destroyed. If there are n players in the game, there are n + 1 possible outcomes:. n outcomes in which some single player wins and one outcome in which all players lose.

The resources of the players can be broken down into four independent resource dimensions. For purposes of the subsequent development of the model, we now present formal definitions of these dimensions.

Definition 1. For any player, X, let D(X) represent the damage player X can inflict on any given player given a successful attack. In terms of the n-uel paradigm, D(X) is the number of markers that player X can destroy in a single turn. It is assumed that $D(X) > 0$, for all X.

Definition 2. For any player, X, let L(X) be the probability that player X will be successful in launching an attack when he chooses to do so. It is assumed that $L(X) > 0$ for all X.

The resources $D(X)$ and $L(X)$ taken together constitute the <u>offensive capability</u> of player X. The two dimensions of resources that constitute the <u>defensive capability</u> of player X can be analogously defined.

Definition 3. For any player, X, let $R(X)$ be the amount of those resources controlled by player X that determine the number of successful attacks he can survive. $R(X)$ designates the number of markers that player X has at any given time. It is assumed that $R(X) > 0$ for all X.

Definition 4. For any player, X, let $I(X)$ be the probability that player X intercepts an attack that is directed at him and thus renders the attack unsuccessful. It is assumed that $I(X) < 1$.

In the laboratory simulation as employed by Cole (1968) and by Cole and Phillips (1967), subjects played the game for a number of turns or trials until at least two of the three were eliminated. Once a player was eliminated he could no longer attack, i.e., if $R(X) = 0$, X was not a participant in the game.

The following section presents a model for relative conflict among three players. This model is intended to account for the interaction in the laboratory's simulation. To the extent that this simulation reflects processes which are operative in other situations, such as international interaction, the model may be useful as a guide to research and the formation of policy.

The Model

Although, in the extended form of the truel, it may be necessary to conduct several trials to determine the outcome, the model which is proposed treats each trial as a game in normal form, that is, each trial is treated as if it were a game in itself. Where it is relevant, however, the number of moves that a participant will last if he is attacked by one of the other participants is used by the model (see definition number 6).

Definitions

The first four definitions have already been given. Definitions 5, 6, and 7 are given below.

Definition 5. Let $S(X,Y)$ be the probability that player X successfully completes an attack on player Y. This is determined by multiplying the probability that player X successfully launches the attack by the probability that player Y does not intercept the attack. $S(X,Y) = [L(X)]\ [1-I(Y)]$.

Definition 6. Let $n(X,Y)$ be the expected number of attacks it takes for player X to eliminate player Y. To determine $n(X,Y)$ it is necessary to compute the ratio of resources controlled by player Y to the amount of damage which player X can inflict times the probability that player X will be successful in inflicting that damage on player Y.

$$n(X,Y) = \frac{R(Y)}{D(X)\ S(X,Y)}$$

Definition 7. Let $P(X,Y)$ = the probability that player X will attack player Y.

Assumptions

(1) One of the factors which influences each participant's attack choice is his vulnerability to each of the other participants. This factor will be referred to as $V(X,Y)$ and is inversely proportional to $n(Y,X)$. Thus, as the number of moves required for player Y to eliminate player X increases, the vulnerability of player X to player Y decreases.
$V(X,Y) = \frac{K_1}{n(Y,X)}$, where K_1 is a constant.

(2) A second factor which influences the participants attack choices is the attack potential of the participants which is referred to as $A(X,Y)$. The attack potential of player X to player Y is directly proportional to the damage player X can inflict on player Y times the probability that player X will be successful in inflicting that damage.
$A(X,Y) = K_2 D(X) S(X,Y)$.

(3) In order to predict the probability that any of the participants will attack one of the other participants it is necessary to compute the relative threats of the participants involved. The simple threat of player X to player Y $[T_o(X,Y)]$ is directly proportional to the attack potential of player X to player Y and the vulnerability of player Y to player X. It is inversely proportional to the vulnerability of player X to player Y.

$$T_o\ (X,Y) = \left[\frac{K_3}{V(X,Y)} \right] V(Y,X) A(X,Y)$$

$$T_o\ (X,Y) = \left[\frac{K_3}{K_1/n(Y,X)} \right] \left[\frac{K_1}{n(X,Y)} \right] K_2 D(X) S(X,Y)$$

$$T_o\ (X,Y) = \left[\frac{n(Y,X) D(X) S(X,Y)}{n(X,Y)} \right] \left[\frac{K_1 K_2 K_3}{K_1} \right]$$

Letting $K = K_2K_3$ we have,

$$T_o\ (X,Y) = \frac{K\left[\frac{R(X)}{D(Y)S(Y,X)}\right] D(X)S(X,Y)}{\frac{R(Y)}{D(X)S(X,Y)}}$$

$$= K\ \frac{R(X)D(X)^2S(X,Y)^2}{R(Y)D(Y)S(Y,X)}$$

Similarly,

$$T_o\ (Y,X) = K\ \frac{R(Y)D(Y)^2S(Y,X)^2}{R(X)D(X)S(X,Y)}$$

$$T_o\ (Z,X) = K\ \frac{R(Z)D(Z)^2S(Z,X)^2}{R(X)D(X)S(X,Z)}$$

$$T_o\ (X,Z) = K\ \frac{R(X)D(X)^2S(X,Z)^2}{R(Z)D(Z)S(Z,X)}$$

$$T_o\ (Y,Z) = K\ \frac{R(Y)D(Y)^2(Y,Z)^2}{R(Z)D(Z)S(Z,Y)}$$

$$T_o\ (Z,Y) = K\ \frac{R(Z)D(Z)^2S(Z,Y)^2}{R(Y)D(Y)S(Y,Z)}$$

(4) The simple probability that player X will attack player Y is equal to the ratio of the threat of player Y to player X [T_o (Y,X)] to the total threat to player X [T_o (Y,X) + T_o (Z,X)].

$$P_o(X,Y) = \frac{T_o\ (Y,X)}{T_o\ (Y,X) + T_o\ (Z,X)}$$

Similarly,

$$P_o\ (X,Y) = \frac{T_o\ (Y,X)}{T_o(X,Y) + T_o(Z,Y)}$$

$$P_o\ (Y,X) = \frac{T_o\ (X,Z)}{T_o(X,Z) + T_o(Y,Z)}$$

$$P_o\ (X,Z) = 1-P_o\ (X,Y)$$

$$P_o\ (Y,X) = 1-P_o\ (Y,Z)$$

$$P_o\ (Z,Y) = 1-P_o\ (Z,X)$$

To clarify the model given above we will discuss an example given by Shubik (1954). Shubik considered a truel in which each of the three participants (A, B, and C) fired one shot at one of the other two participants. The strength of each participant was determined by his probability of hitting his target. The respective strengths were: A = 0.8; B = 0.7; and C = 0.6. Shubik considered the case in which successive firing order was randomly determined. There were six equally probable firing orders. Assuming that the stronger of the two attack alternatives (that is, the participant who posed the greatest threat to a person's survival) would be attacked with probability one by each of the participants, Shubik determined that, averaged over the six firing orders, A's chances of survival were 0.260, B's chances of survival were 0.488, and C's chances of survival were 0.820. Thus, the poorest shot had the best chance to survive. This phenomena was referred to as "strength through weakness" by Shubik. We will apply our model to the Shubik example with one modification; the firing order will not be considered due to our assumption of simultaneous attacks rather than the successive attacks assumed by Shubik.

The first step in applying our model to Shubik's example is to determine the offensive and defensive capabilities for each of the participants. Shubik's "probability of hitting the target" is equivalent to the launch probability in the present model, so the following values are appropriate.

Person A	Person B	Person C
D(A) = 1	D(B) = 1	D(C) = 1
L(A) = .8	L(B) = .7	L(C) = .6
R(A) = 1	R(B) = 1	R(B) = 1
I(A) = 0	I(B) = 0	I(B) = 0

From the above definitions we computed the following probabilities of each participant successfully completing an attack on each of the other participants.

$$S(A, B) = S(A, C) = .8$$
$$S(B, A) = S(B, C) = .7$$
$$S(C, A) = S(C, B) = .6$$

Using the formulas which were given in Assumption 3, the following simple threats were computed.

$$T_o\ (A, B) = K\ \frac{R(A)\ D(A)^2\ S(A,B)^2}{R(B)\ D(B)\ S(B,A)}$$

$$= \frac{(1)\ (1)^2\ (.8)^2}{(1)\ (1)\ (.7)}\ K$$

$$= \frac{.64}{.7}\ K$$

$$= .915\ K$$

$$T_o\ (A, C) = 1.068\ K$$

$$T_o\ (B, A) = .613\ K$$

$$T_o\ (B, C) = .817\ K$$

$$T_o\ (C, A) = .450\ K$$

$$T_o\ (C, B) = .514\ K$$

The simple probabilities of attack as computed from the formulas in Assumption 4 are:

$$P_o\ (A, B) = \frac{T_o(B,A)}{T_o(B,A) + T_o(C,A)}$$

$$= \frac{.613K}{.613K + .450K}$$

$$= .577$$

$$P_o(A, C) = .423$$

$$P_o(B,A) = .641$$

$P_o(B, C) = .359$

$P_o(C, A) = .567$

$P_o(C, B) = .433$

The probability of survival obviously depends upon the attack probabilities. If we denote the probability of a successful attack on X by Y as

$$P\ (Y,X)$$

then

$$P(X \text{ survives}) = 1 - [P'(Y,X) + P'(Z,X) - P'(Y,X)P'(Z,X)]$$

Thus:

$P(A \text{ surviving}) = 1 - [(.449) + (.340) - (.148)] = .359$

$P(B \text{ surviving}) = 1 - [(.462) + (2.60) - (.120)] = .398$

$P(C \text{ surviving}) = 1 - [(.338) + (.251) - (.085)] = .496$

The model concurs with the prediction of "strength through weakness" as proposed by Shubik (1954), however, the "strength through weakness" effect as predicted by the model is much weaker than was predicted by Shubik. The difference between Shubik's predictions and the model's predictions follows from the differential probabilities of attacking the person who poses the greatest threat to one's survival as well as from the assumption of simultaneous rather than successive attacks. Shubik assumes that each person will attack the person who poses the greatest threat to his survival with probability one. Our model, on the other hand, assumes that each person will consider the contribution to the total threat against him which is associated with each of the other participants, and that the probability of attacking a given participant varies directly with the proportion of the total threat contributed by that participant.

As they are stated, both Shubik's interpretation and the model's interpretation of strategy selection in the truel assume a rational approach. The difference between the two interpretations is merely that each assumes a different decision rule. Both are subject to the criticism that the decision rule that they propose oversimplifies the situation considerably. Realizing the appropriateness of this criticism and that in fact such simple decision rules overlook the psychological processes present within the situation, we extended the model to include the cognitive processes employed in strategy selection.

The extension of the model incorporates the probability of being attacked into the determination of the threat. Thus, the simple threat as proposed previously is modified by the subjective probability of being attacked. This modification is given in definition 8 and assumption 5 below.

Definition 8. For every pair of players, X and Y, let $\Phi_{y,o}(X,Y)$ be Y's subjective probability of being attacked by X.

Assumption 5. The (once) revised threat of player X to player Y is equal to the product of the simple threat of player X to player Y and player Y's subjective probability of being attacked by player X. Simbolically:

$$T_1(X,Y) = T_o(X,Y)\,\Phi_{y,o}(X,Y)$$

The introduction of subjective probabilities raises the question of how those subjective probabilities are determined. This question can be partitioned into two questions: (1) how does a player determine subjective probabilities for his own prospective action and (2) how does a player determine subjective probabilities for the prospective action of others? Our answers to these questions are given in assumptions 6 and 7 below.

Assumption 6. Any player, X, knows, without error, his own attack probabilities.

Thus:

$$\phi_{X,o}(X,Y) = P_o(X,Y).$$

Assumption 7. Any player expects that for any two players, X and Y, the probability that Y attacks X will be directly proportionate to the probability that X attacks Y. In other words, each player assumes reciprocity with respect to attacks between any two players. From the standpoint of player X, this means

$$\phi_{X,o}(Y,X) = \phi_{X,o}(X,Y)\ \alpha_X(Y,X),$$

$$\phi_{X,o}(Z,X) = \phi_{X,o}(X,Z)\ \alpha_X(Z,X),$$

$$\phi_{X,o}(Y,Z) = \phi_{X,o}(Z,Y)\ \alpha_X(Y,Z),$$

where $\alpha_X(Y,X)$, $\alpha_X(Z,X)$ and $\alpha_X(Y,Z)$ are the coefficients of proportionality.

It can be shown that so long as the objective attack probabilities are all less than one and greater than zero, the coefficients of proportionality can be chosen so as to make the subjective probabilities equal to the objective attack probabilities. It is of interest, however, to make a somewhat different assumption about these coefficients.

Assumption 8. For any player, X, the subjective probability of being attacked by any other player, Y, is equal to the probability with which X will attack Y. Clearly, the implications for the coefficients of proportionality are

$$\alpha_X(Y,X) = \alpha_X(Z,X) = 1$$

Assumption 8 has consequences for the coefficient of proportionality $\alpha_x(Y,Z)$. Before discussing these consequences, it is necessary to make clear the meaning of a coefficient of proportionality. If, for example, $\alpha_x(Y,Z) = 1$, then player X expects Y and Z to play a tit-for-tat strategy against each other, that is, Y and Z may be said to behave according to some rule or norm of <u>perfect reciprocation</u>. If $\alpha_x(Y,Z) < 1$, then player X assumes that player Y will <u>under-reciprocate</u> attacks from Z, that is, Y will attack Z less frequently than Z attacks Y. On the other hand, if $\alpha_x(Y,Z) > 1$, then player X assumes <u>over-reciprocation</u> by Y against Z. Thus, assumption 8 simply reflects the hypothesis that any player expects perfect reciprocation between himself and any other player. Theorem 1 details the determinants of $\alpha_x(Y,Z)$.

Theorem 1. The coefficient of proportionality, $\alpha_x(Y,Z)$, is equal to the ratio of the probability that player X attacks player Z to the probability that player X attacks player Y. That is

$$\alpha_x(Y,Z) = \frac{P_o(X,Z)}{P_o(X,Y)} .$$

Proof: From assumption 7 we have

(1) $\phi_{x,o}(Y,Z) = \phi_{n,o}(Z,Y)\ \alpha_x(Y,Z)$ which implies

(2) $\alpha_x(Y,Z) = \dfrac{\phi_{x,o}(Y,Z)}{\phi_{x,o}(Z,Y)}$.

Since $\phi_{x,o}(Y,Z) = 1 - \phi_{x,o}(Y,X)$ and $\phi_{x,o}(Z,Y) = 1 - \phi_{x,o}(Z,X)$ we have:

(3) $\alpha_x(Y,Z) = \dfrac{1-\phi_{x,o}(Y,X)}{1-\phi_{x,o}(Z,X)}$.

From assumption 7, this becomes

(4) $\alpha_x(Y,Z) = \dfrac{1-\phi_{x,o}(X,Y)\ \alpha_x(Y,X)}{1-\phi_{x,o}(X,Z)\ \alpha_x(Z,X)}$,

which, in virtue of assumption 8 simplifies to

$$(5)\quad \alpha_X(Y,Z) = \frac{1-\phi_{x,o}(X,Y)}{1-\phi_{x,o}(X,Z)} .$$

Assumption 6 allows us to replace subjective probabilities with objective probabilities as follows:

$$(6)\quad \alpha_X(Y,Z) = \frac{1-P_o(X,Y)}{1-P_o(X,Z)} = \frac{P_o(X,Z)}{P_o(X,Y)}$$

and the theorem is proved.

Assumptions 5 - 8 allow us to caluclate revised threat values. Since those values differ to some extent from the simple threat values, and since the attack probabilities were determined by the simple threat values, it is necessary to derive revised attack probabilities. These revised probabilities, although dependent upon the revised threat values, can be expressed in terms of simple threats, as in theorem 2.

Theorem 2. The (once) revised probability of player X attacking player Y, denoted $P_1(X,Y)$ is given by the following equation:

$$P_1(X,Y) = \frac{T_o(Y,X)^2}{T_o(Y,X)^2 + T_o(Z,X)^2}$$

Proof: By extension of assumption 4

$$(1)\quad P_1(Z,Y) = \frac{T_1(Y,X)}{T_1(Y,X) + T_1(Z,X).}$$

From assumption 5 we can re-write expression (1) as follows:

$$(2)\quad P_1(X,Y) = \frac{T_o(Y,X)\,\phi_{x,o}(Y,X)}{T_o(Y,X)\,\phi_{x,o}(Y,X) + T_o(Z,X)\,\phi_{x,o}(Z,X)}$$

which, in virtue of assumptions 6, 7, and 8 yields

$$(3)\quad P_1(X,Y) = \frac{T_o(Y,X)\ P_o(X,Y)}{T_o(Y,X)\ P_o(X,Y) + T_o(Z,X)\ P_o(X,Z)} .$$

Assumption 4 gives the simple attack probabilities in terms of simple threats. When those are substituted here we have:

$$(4)\quad P_1(X,Y) = \frac{\dfrac{T_o(Y,X)^2}{T_o(Y,X) + T_o(Z,X)}}{\dfrac{T_o(Y,X)^2}{T_o(Y,X) + T_o(Z,X)} + \dfrac{T_o(Z,X)}{T_o(Y,X) + T_o(Z,X)}},$$

which reduces to:

$$(5)\quad P_1(X,Y) = \frac{T_o(Y,X)^2}{T_o(Y,X)^2 + T_o(Z,X)^2},$$

and the theorem is proved.

The model can be further extended along the same lines. The rationale for this extension is the ascription to the participants in uelative conflict of an active reflective process. This process results in a modification of the subjective probabilities which, in turn, modify the threats and thus the attack probabilities. In order to achieve this extension, we restate assumptions 4, 5, 6, and 7 below.

Assumption 4^*. The probability that player X will attack player Y is equal to the ratio of the threat of player Y to player X to the total threat to player X. Symbolically:

$$P_n(X,Y) = \frac{T_n(Y,X)}{T_n(Y,X) + T_n(Z,X)}$$

Assumption 5^*. The nth revised threat of player X to player Y is equal to the product of the (n-1)th revised threat of player X to player Y and player Y's (n-1)th revision of his subjective probability of being attacked by player X. Symbolically:

$$T_n(X,Y) = T_{n-1}(X,Y)\,\phi_{Y,n-1}(X,Y).$$

Assumption 6*. Any player, X, knows his own attack probabilities.

Thus $\quad \phi_{X,n}(X,Y) = P_n(X,Y)$

Assumption 7*. Each player assumes reciprocity with respect to attacks between any two players. From the standpoint of player X, this means

$$\phi_{X,n}(Y,X) = \phi_{X,n}(X,Y)\alpha_X(Y,X),$$

$$\phi_{X,n}(Z,X) = \phi_{X,n}(X,Z)\alpha_X(Z,X),$$

$$\phi_{X,n}(Y,Z) = \phi_{X,n}(Z,Y)\alpha_X(Y,Z).$$

Since it will be useful to have access to subjective-objective probability conversion expressions, we provide them below without proof.

Lemma 1. Conversion expressions for subjective probabilities are

$$\phi_{X,n}(X,Y) = P_n(X,Y)$$

$$\phi_{X,n}(Y,X) = P_n(X,Y)$$

$$\phi_{X,n}(X,Z) = P_n(X,Z)$$

$$\phi_{X,n}(Z,X) = P_n(X,Z).$$

We now state a theorem which allows the nth revised attack probabilities to be calculated from the simple threats.

Theorem 3. The nth revised probability of player X attacking player Y is

$$P_n(X,Y) = \frac{T_o(Y,X)^{2^n}}{T_o(Y,X)^{2^n} + T_o(Z,X)^{2^n}} .$$

Proof (by induction): It has been shown in theorem 2 that the general formula holds for one reflective cycle. Assume it is true for n-1 reflective cycles, that is:

$$(1)\quad P_{n-1}(X,Y) = \frac{T_{n-1}(Y,X)}{T_{n-1}(Y,X)+T_{n-1}(Z,X)} = \frac{T_o(Y,X)^{2^{n-1}}}{T_o(Y,X)^{2^{n-1}}+T_o(Z,X)^{2^{n-1}}}$$

From assumption 5 we have

$$(2)\quad T_n(Y,X) = T_{n-1}(Y,X)\ \phi_{y,n-1}(Y,X) \text{ and}$$

$$T_n(Z,X) = T_{n-1}(Z,X)\ \phi_{y,n-1}(Z,X).$$

Expression (2) can be rewritten as

$$(3)\quad T_n(Y,X) = T_{n-1}(Y,X)\ P_{n-1}(X,Y)$$

$$T_n(Z,X) = T_{n-1}(Z,X)\ P_{n-1}(X,Z) \text{ using Lemma 1.}$$

Combining expressions (1) and (3) into the form of assumption 4 yields

(4)

$$P_n(X,Y) = \frac{T_o(Y,X)^{2^{n-1}} \left\{ \dfrac{T_o(Y,X)^{2^{n-1}}}{T_o(Y,X)^{2^{n-1}}+T_o(Z,X)^{2^{n-1}}} \right\}}{T_o(Y,X)^{2^{n-1}} \left(\dfrac{T_o(Y,X)^{2^{n-1}}}{T_o(Y,X)^{2^{n-1}}+T_o(Z,X)^{2^{n-1}}} \right) + T_o(Z,X)^{2^{n-1}} \left(\dfrac{T_o(Z,X)^{2^{n-1}}}{T_o(Y,X)^{2^{n-1}}+T_o(Z,X)^{2^{n-1}}} \right)}$$

Expression (4) can be simplified to

(5)

$$P_n(X,Y) = \frac{[T_o(Y,X)^{2^{n-1}}]\ [T_o(Y,X)^{2^{n-1}}]}{[T_o(Y,X)^{2^{n-1}}]\ [T_o(Y,X)^{2^{n-1}}] + [T_o(Z,X)^{2^{n-1}}]\ [T_o(Z,X)^{2^{n-1}}]}$$

or, alternatively

$$(6)\quad P_n(X,Y) = \frac{[T_o(Y,X)^{2^{n-1}}]^2}{[T_o(Y,X)^{2^{n-1}}]^2 + [T_o(Z,X)^{2^{n-1}}]^2}$$

which is just

$$(7)\quad P_n(X,Y) = \frac{T_o(Y,X)^{2^n}}{T_o(Y,X)^{2^n} + T_o(Z,X)^{2^n}}$$

and the theorem is proved.

We shall be interested in obtaining asymptotic values of $P_n(X,Y)$ as the active reflective process continues indefinitely. To facilitate this result, we state the following corollary:

Corollary 1. An alternative expression for $P_n(X,Y)$ is given by

$$P_n(X,Y) = \frac{1}{1 + \left[\frac{T_o(Z,X)}{T_o(Y,X)}\right]^{2n}} \quad .$$

Theorem 4. As the active reflective process continues indefinitely, any player, X, attacks that player who constitutes the greater simple threat to him with probability 1 and that player who constitutes the lesser simple threat to him with probability zero. If both of X's opponents are equal in simple threat, he attacks each with probability 1/2.

Proof: There are three cases.

Case 1. Assume that player Z constitutes a greater simple threat to player X than does player Y, i.e.,

$$T_o(Z,X) > T_o(Y,X).$$

In this case, we know that

$$\frac{T_o(Z,X)}{T_o(Y,X)} > 1 \quad .$$

Taking the limit of $P_n(X,Y)$ as n grows indefinitely large we obtain

$$\lim_{n \to \infty} P_n(X,Y) = \frac{1}{1 + \lim_{n \to \infty} \left[\frac{T_o(Z,X)}{T_o(Y,X)} \right]^{2^n}} = 0$$

Case II. Assume that player Y constitutes a greater simple threat to player X than does player Z. Under this assumption it must be true that

$$\frac{T_o(Z,X)}{T_o(Y,X)} < 1.$$

Taking the limit of $P_n(X,Y)$ under this assumption yields

$$\lim_{n \to \infty} P_n(X,Y) = \frac{1}{1 + \lim_{n \to \infty} \left[\frac{T_o(Z,X)}{T_o(Y,X)} \right]^{2^n}} = 1.$$

Case III. Assume that $T_o(Z,X) = T_o(Y,X)$. This implies that

$$\frac{T_o(Z,X)}{T_o(Y,X)} = 1.$$

If we take the limit of $P_n(X,Y)$ we have

$$\lim_{n \to \infty} P_n(X,Y) = \frac{1}{1 + \lim_{n \to \infty} \left[\frac{T_o(Z,X)}{T_o(Y,X)} \right]^{2^n}} = 1/2$$

and the theorem is proved.

Experiment 1

The initial test of the model employed the truel as an experimental paradigm because it is the simplest nontrivial n-uel. Three experimental manipulations were conducted. The first manipulation held R(X), L(X), and I(X) constant and varied D(X) such that situations ranging from a situation in which D(A) = D(B) = D(C)--the all equal point--to a situation in which one player had complete control of the outcome--the dictator situation--were tested. The second manipulation held D(X), L(X), and I(X) constant and varied R(X) such that situations ranging from the all equal point to the dictator situation were tested. The third manipulation held D(X), R(X), and I(X) constant and varied L(X) such that situations ranging from the all equal point to a situation in which one player had an obvious advantage with respect to the probability of successfully completing an attack were examined. Due to the probabilistic nature of the third manipulation it was not feasible to manipulate the situation such that one player had complete control over the outcome. The three manipulations and the procedures and results associated with each one will be presented separately as experiments 1a, 1b, and 1c.

Experiment 1a

Subjects. Forty-five male and forty-five female undergraduate volunteers from an introductory psychology class at Michigan State University were the subjects in this experiment. They received two extra credit points in the course for their participation.

Apparatus. The apparatus consisted of 90 white poker chips and three wooden tokens one marked A, one marked B, and one marked C.

Special attack choice slips were provided and clipboards were provided to allow attack choices to remain secret until all three players had made their attack. A cylindrical urn that measured 2 3/8 inches in diameter and 4 inches tall and that had an internal baffle was used for the random selection of player position for each game.

Procedure. Fifteen all male and fifteen all female triads participated in the experiment. Each triad played five games in which D(X) for each of the three participants (Player A, Player B, and Player C) was assigned as indicated in Table 1. All of the games were played in a

Insert Table 1 about here

face-to-face situation and the order in which the games were played was randomly determined for each triad. The position that each subject played was determined randomly for each game. Before each game, each subject drew a token (marked either A, B, or C) from an urn and played the game in the position indicated on that token.

The structure of the basic game was as follows. Each of the players began each game with 30 poker chips [R(A) = R(B) = R(C) = 30]. The probability of intercepting an attack [I(X)] was 0.0 and the probability of launching an attack [L(X)] was 1.0 for all of the players in all of the games. The rules of the game required that each player remove a given number of chips (see Table 1) from one of the other two players on each move. To remove chips each player circled the letter of the player he wished to attack on an attack choice slip and passed it to the experimenter. After the experimenter had received attack choice slips from all three players on each move, the players were told who had

TABLE 1

D(A), D(B), and D(C) for the Five Game Types in Experiment Ia

	Game Type				
	1	2	3	4	5
D(A)	6	7	8	9	11
D(B)	6	6	6	6	6
D(C)	6	5	4	3	1

*Note.--The five distributions of D(X) were selected to regulate game length and to test the model in situations ranging from the all equal situation--the 6/6/6 game--to a situation in which one player had dictatorial powers--the 11/6/1 game.

attacked whom and the number of chips remaining was appropriately adjusted. Thus, the moves in the game were simultaneous. The chips that were taken away, were taken out of the game and did not belong to any of the players. The winner of each game was that player who had points remaining when the other players' points were gone, that is he was the sole survivor. If there was no sole survivor, there was no winner.

Each of the subjects was required to fill out a pre-game questionnaire before each of the five games. The form of the questionnaire follows:

Game Experiment
Pre-game Questionnaire

Before you draw to determine which of you will play in which position, I would like you to answer the following question.

1. Which player would you choose to be if you had your choice?

A B C No preference
(circle one)

In addition, each of the players filled out a post-game questionnaire following the five games. The following nine questions were asked on that questionnaire.

1. Did you know anything about the experiment before you came in the room? If so, what?

2. How would you rate the length of the game?

1 2 3 4 5
too short too long

3. How interesting was the game?

1 2 3 4 5
very dull very interesting

4. Was the game fair? If not, please state your reasons.

5. How easy was it to understand the rules of the game?

1 2 3 4 5
very easy very difficult

6. How hard did you try to win?

1 2 3 4 5
not at all very hard

7 Did you know either of the other two players before today? If you did, how well did you know him?

1 2 3 4 5
not at all very hard

8. Which player would you rather be?

A B C No Preference
(circle one)

9. What did you think I was trying to study with this experiment?

After the subjects had completed the post-game questionnaire, they were thoroughly debriefed.

Results. As a preliminary test of the model, we will concentrate on the initial attack. Thus, until it is stated otherwise, all data discussed will be attacks on the first move of the game. Figure 1 indicates the predicted probabilities of each player attacking the stronger of his two opponents as a function

Insert Figure 1 about here

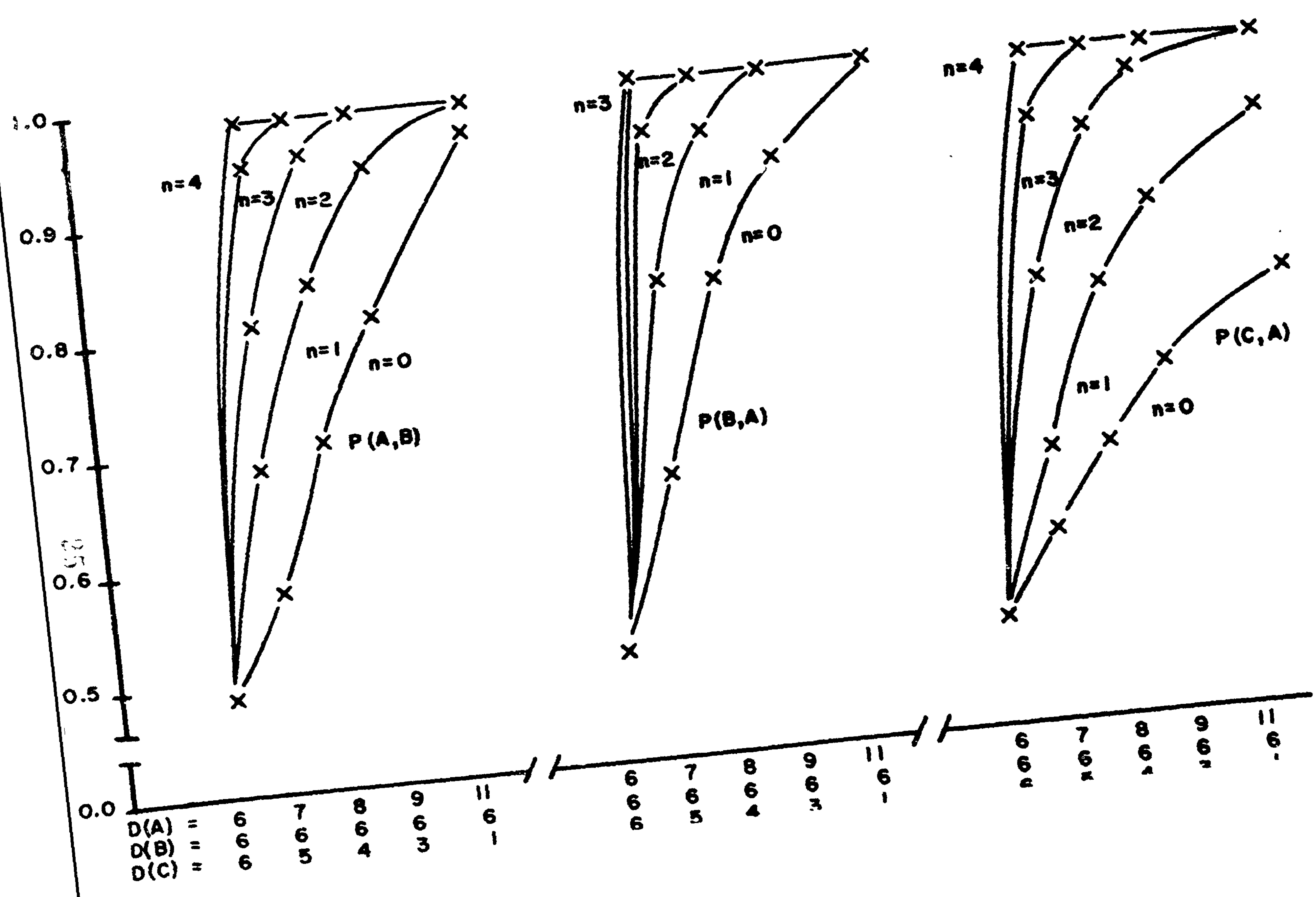
of D(X) for n reflective cycles. It is obvious from Figure 1 that the predicted probabilities of attack asymptote at 1.0 after a small number of reflective cycles.

Figure 2 reports the observed probability of attack as compared to the predicted probability of attack as a function of D(X) on the initial trial of all game types.

Insert Figure 2 about here

Figure 1.

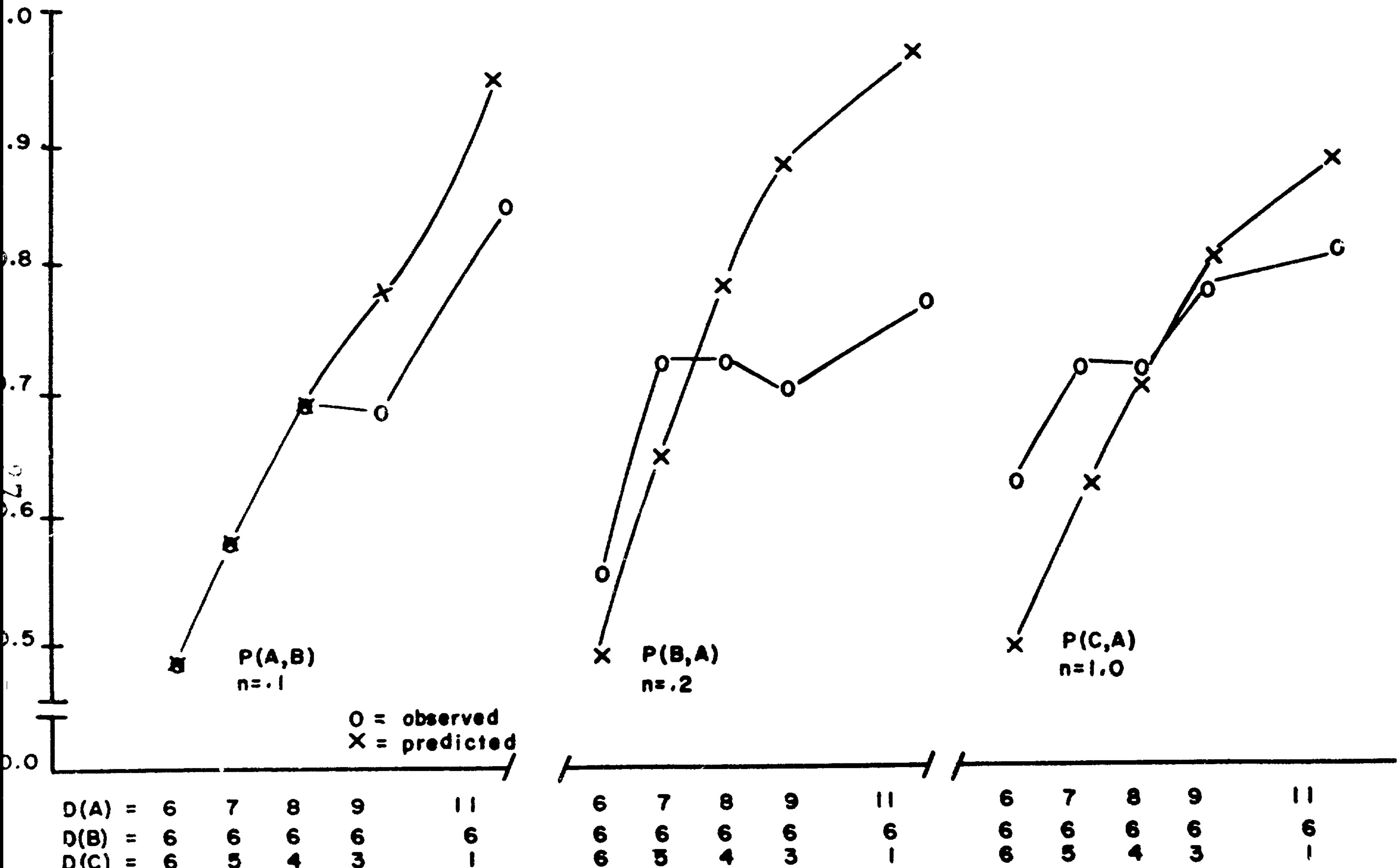
Predicted probabilities of each player attacking the stronger of his two opponents as a function of D(X) for 'n' reflective cycles.



P (A,B)
P(B,A)
P (C, A)
n=0
n=1
n=2
n=3
n=4
1.0
0.9
0.8
0.7
0.6
0.5
0.0
D(A) = 6 7 8 9 11
D(B) = 6 6 6 6 6
D(C) = 6 5 4 3 1

Figure 2.

Observed and predicted probabilities of each player attacking the stronger of his two opponents as a function of D(X) and 'n' reflective cycles.



.0
.9
.8
.7
.6
.5
0.0
P(A,B)
n=.1
P(B,A)
n=.2
P(C,A)
n=1.0
O = observed
X = predicted
D(A) = 6 7 8 9 11
D(B) = 6 6 6 6 6
D(C) = 6 5 4 3 1

The predicted probabilities of attack were obtained using the average number of reflective cycles associated with Players A, B, and C for game types two through five. The n's were computed by using Theorem 3 and substituting the observed probability of attack for the predicted probability of attack. Since the appropriate simple threats could be computed, the matter of solving for n, the only parameter, was easily performed. Such a procedure may seem inappropriate, because we are forced to assume that a player can use a fraction of a reflective cycle. However, it is reasonable for some subset of the players to utilize j reflective cycles while some other subset of players considers i reflective cycles. The result is some number of reflective cycles n which is composed of the weighted average of j and i.

Examination of figure 2 indicates that the model does not accurately predict attack choices on the initial move of a truel in which D(X) is manipulated. However, there are indications that the assumptions underlying the model may in fact be acceptable. Further discussion of the inadequacies of the model will be discussed in conjunction with the results of the experiments lb and lc. We will now turn to the data obtained from all of the moves in each game type.

Figure 3 reports the observed and predicted probabilities of each player attacking the stronger of his two opponents as a function of D(X) and n reflective cycles for each move of game types 6/6/6, 7/6/5, 8/6/4, 9/6/3, and 11/6/1 respectively. The number of relfective cycles associated with each player in each game type was computed by determining the exact number of reflective cycles which predicted each move for each player in each game type and determining the mean number of reflective cycles for each player across all moves for each game type. Any move with less than twenty subjects (i.e., N < 20) was excluded. The product-moment correlations between the predicted and observed values are presented in Table 2.

Insert Figure 3 and Table 2 about here

Experiment 1b

Subjects. Ninety male and ninety female undergraduate students from an introductory psychology class at Michigan State University were the subjects in this experiment. They received extra credit in the course for their participation.

Apparatus. The apparatus for this experiment was identical to the apparatus used in experiment 1a.

Procedure. Thirty all male and thirty all female triads were formed. Each triad played four games in which R(X) for each of the three participants (Player A, Player B, and Player C) was assigned as indicated in Table 3. Fifteen male triads and fifteen female triads played game types 1, 3, 5, and 7;

Insert Table 3 about here

fifteen male triads played game types 1, 2, 4, and 6; fifteen female triads played game types 2, 4, 6, and 8 in those orders. As was the case in experiment 1a, all of the games were played in a face-to-face situation. The position that each subject played in each game was randomly determined by the same method used in experiment 1a.

The structure of the basic game was as follows. Each of the players began each game with R(X) determined by the game type that they were playing (see Table 3). The probability of launching an attack [L(X)] was 1.0 for all of the players in all of the games. Except for the fact that each player removed only one chip on each move [D(A) = D(B) = D(C) = 1], the rules of the game were

Figure 3.

Observed and predicted probabilities of each player attacking the stronger of his two opponents as a function of move number and 'n' reflective cycles with the 6/6/6 [D(X)] game type appearing in "a", 7/6/5 in "b", 8/6/4 in "c", 9/6/3 in "d", and 11/6/1 in "e".

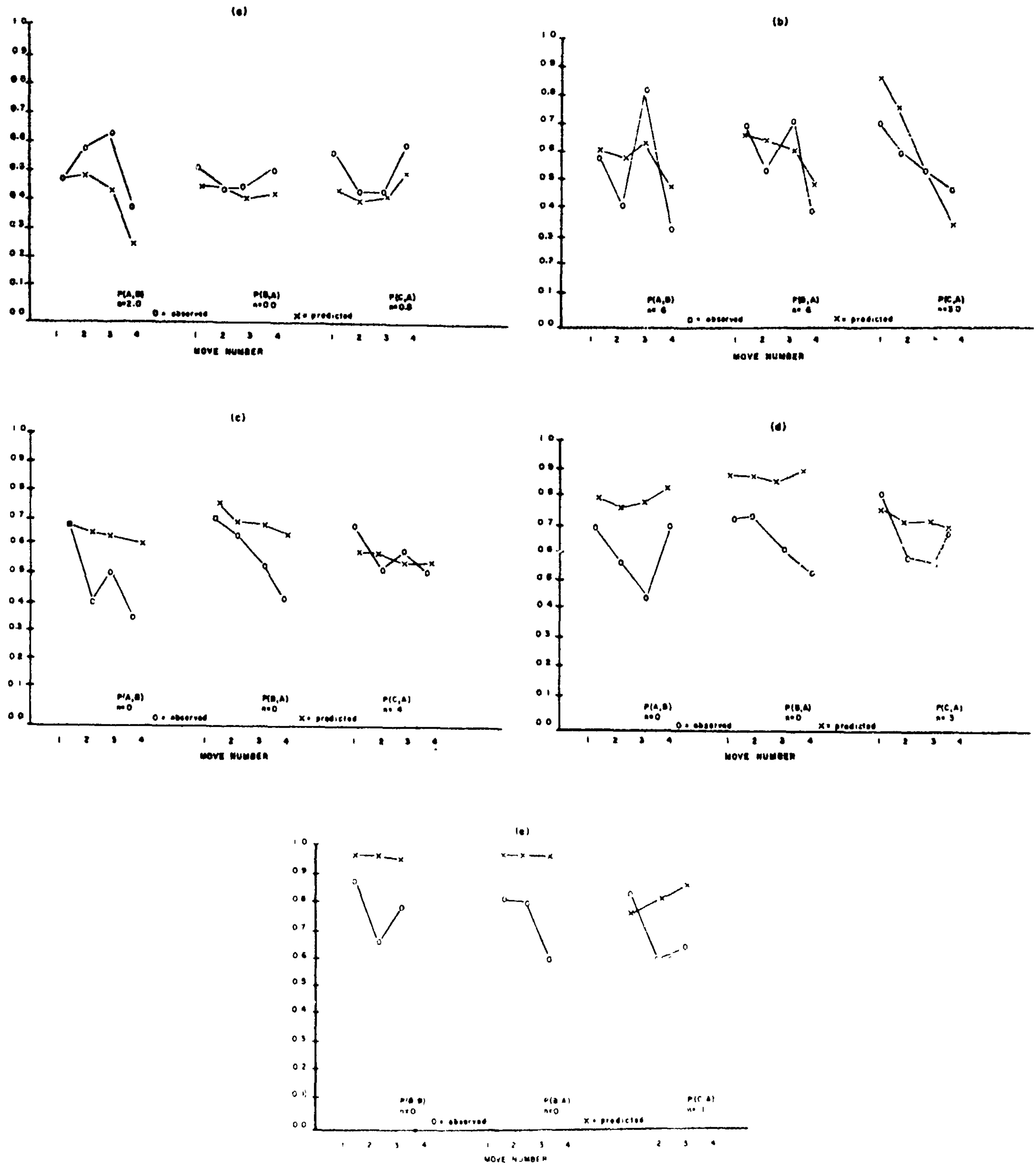

(a)
P(A,B) n=2.0
P(B,A) n=0.0
P(C,A) n=0.5
(b)
P(A,B) n=6
P(B,A) n=6
P(C,A) n=3.0
(c)
P(A,B) n=0
P(B,A) n=0
P(C,A) n=4
(d)
P(A,B) n=0
P(B,A) n=0
P(C,A) n=3
(e)
O = observed
X = predicted
MOVE NUMBER

TABLE 2

Prodict-Moment Correlations between Predicted and Observed Probabilities of Each Player Attacking the Stronger of his Two Opponents for Each Game Type in Experiment 1b

Player	Game Type				
	1	2	3	4	5
A	.72	.75	.98	.88	.42
B	.32	.82	.97	-.27	.70
C	.83	.93	.65	-.24	-.70

TABLE 3

R(A), R(B), and R(C) for the Eight Game Types in Experiment 1b

	Game Type							
	1	2	3	4	5	6	7	8
R(A)	9	10	11	12	13	14	15	16
R(B)	9	9	9	9	9	9	9	9
R(C)	9	8	7	6	5	4	3	2

*Note.--The eight distributions of R(X) were selected to regulate game length and to test the model in situations ranging from the all equal situation--the 9/9/9 game--to a situation in which one player had dictatorial powers--the 16/9/2 game.

identical to the rules governing the game in experiment 1a.

Results. As was the case in experiment 1a, in this experiment, we first considered only attack data from the initial move of the game. Figure 4 indicates the predicted probabilities of each player attacking the stronger of his two opponents as a function of R(X) for n reflective cycles.

Insert Figure 4 about here

Figure 5 reports the observed probability of attack as compared to the predicted probability of attack as a function of R(X). The number of reflective

Insert Figure 5 about here

cycles was computed in the same manner that it was computed in experiment 1a. The n's that were computed for each player position in each of the game types two through eight, were summed and divided by seven. The resulting mean n for each player position was used to determine the predicted probabilities of attack in all game types.

Figure 6 reports the observed and predicted probabilities of each player attacking the stronger of his two opponents as a function of R(X)

Insert Figure 6 about here

and n reflective cycles for each move of game types 9/9/9, 10/9/8, 11/9/7, 12/9/6, 13/9/5, 14/9/4, and 15/9/3 respectively. The number of reflective cycles was computed as it was in experiment 1a. Any move with N < 20 was excluded. Table 4 presents the product-moment correlations between the observed and predicted probabilities.

Figure 4.

Predicted probabilities of each player attacking the stronger of his two opponents as a function of R(X) for 'n' reflective cycles.

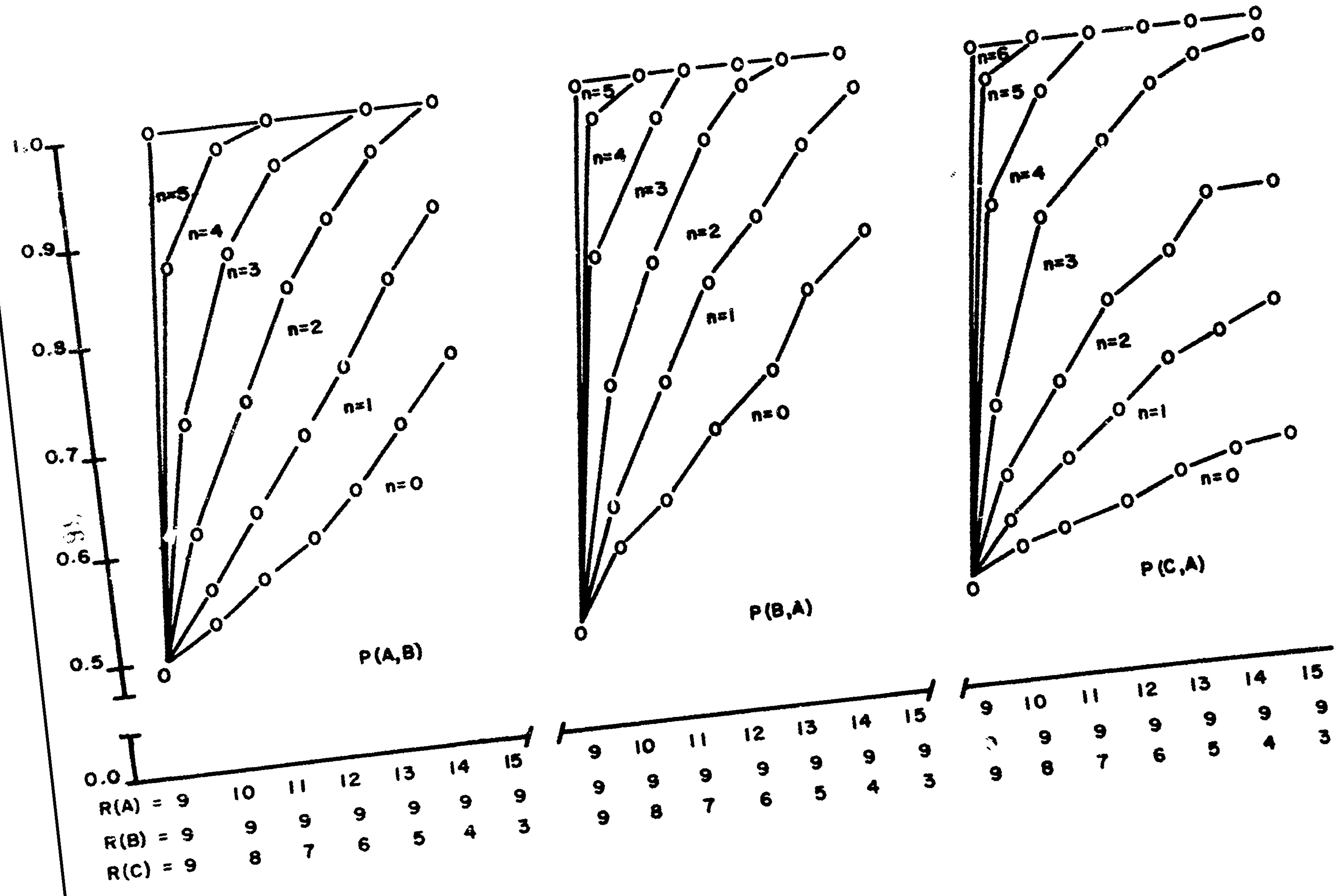

1.0
0.9
0.8
0.7
0.6
0.5
0.0
P(A,B)
P(B,A)
P(C,A)
n=0
n=1
n=2
n=3
n=4
n=5
n=6
R(A) = 9 10 11 12 13 14 15
R(B) = 9 9 9 9 9 9 9
R(C) = 9 8 7 6 5 4 3

Figure 5.

Observed and predicted probabilities of each player attacking the stronger of his two opponents as a function of R(X) with 'n' reflective cycles.

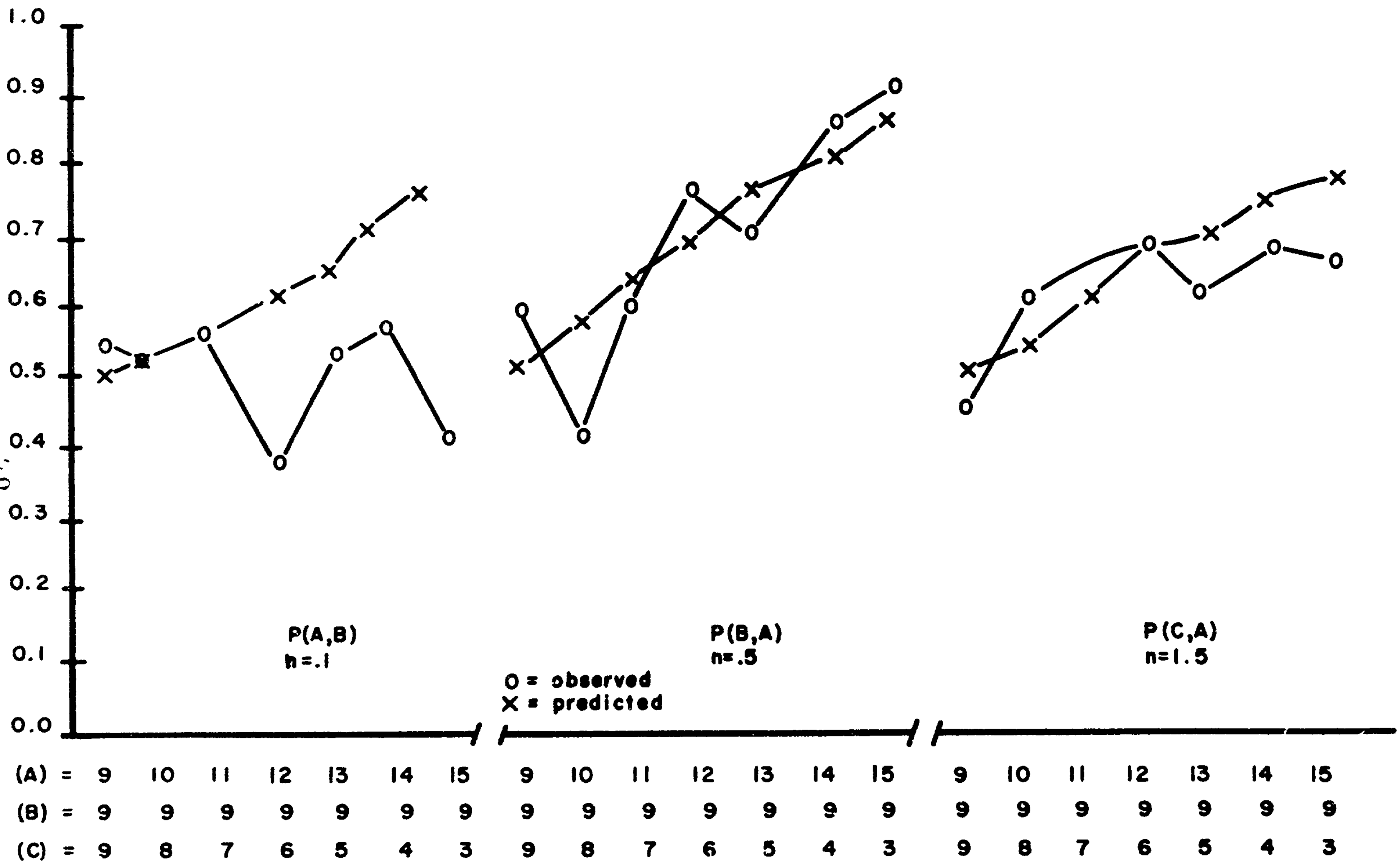
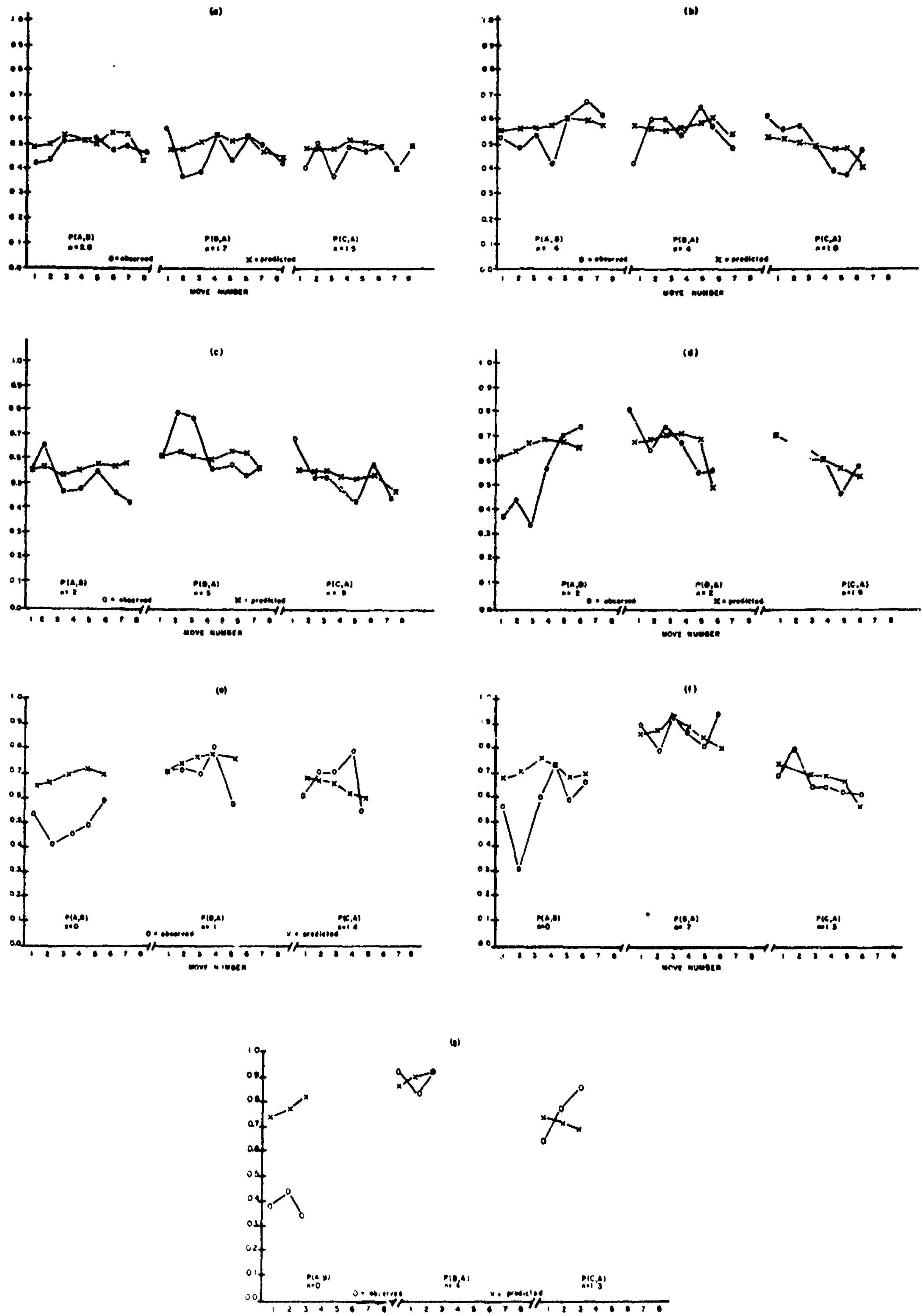

1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
P(A,B)
h=.1
P(B,A)
n=.5
P(C,A)
n=1.5
O = observed
X = predicted
(A) = 9 10 11 12 13 14 15 9 10 11 12 13 14 15 9 10 11 12 13 14 15
(B) = 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9
(C) = 9 8 7 6 5 4 3 9 8 7 6 5 4 3 9 8 7 6 5 4 3

Figure 6.

Observed and predicted probabilities of each player attacking the stronger of his two opponents as a function of move number and 'n' reflective cycles with the 9/9/9 [R(X)] game type appearing in "a", 10/9/8 in "b", 11/9/7 in "c", 12/9/6 in "d", 13/9/5 in "e", 14/9/4 in "f", and 15/9/3 in "g".


(a)
(b)
(c)
(d)
(e)
(f)
(g)
P(A,B)
P(B,A)
P(C,A)
O = observed
X = predicted
MOVE NUMBER

Insert Table 4 about here

Experiment 1c

Subjects. Forty-five male and forty-five female undergraduate volunteers from an introductory psychology course at Michigan State University were the subjects in this experiment. They received extra credit in the course for their participation.

Apparatus. The basic apparatus for this experiment was identical to the apparatus used in experiments 1a and 1b. In addition, due to the probabilistic nature of the present experiment, three sampling urns were used. The sampling urns were cylindrical and measured 1 1/4 inch in diameter and 2 1/4 inches in height. Each urn contained a total of 10 balls approximately 1/8 inch in diameter. Each urn had a clear plastic bubble on top which was constructed so that when the urn was turned upside down, one of the ten balls would fall into the bubble.

Procedure. Fifteen all male and fifteen all female triads participated in the experiment. Each triad played five games in which L(X) for each of the three participants (Player A, Player B, and Player C) was assigned as indicated in Table 5. All of the games were played in a face-to-face situation and the order in which the games were played was randomly determined for each trial. The position that each subject played was determined by the same method

Insert Table 5 about here

that was used in experiments 1a and 1b.

The structure of the basic game was as follows. Each of the players began

TABLE 4

Product-Moment Correlations between Predicted and Observed Probabilities of Each Player Attacking the Stronger of his Two Opponents for Each Game Type in Experiment 1b

Player	Game Type						
	1	2	3	4	5	6	7
A	.56	.70	.06	.51	-.04	.46	-.68
B	.34	.66	.16	.32	.07	-.14	.05
C	.57	.53	.71	.78	-.07	.81	-.78

TABLE 5

L(A), L(B), and L(C) for the Five Game Types in Experiment 1c

	Game Type				
	1	2	3	4	5
L(A)	.6	.7	.8	.9	1.0
L(B)	.6	.6	.6	.6	.6
L(C)	.6	.5	.4	.3	.2

*Note.--The five distributions of L(X) were selected to allow a test of the model in a variety of situations in which the predicted outcomes are similar to the predicted outcomes in experiments 1a and 1b.

each game with 6 poker chips [R(A) = R(B) = R(C) = 6]. The probability of intercepting an attack [I(X)] was 0.0 and each player could remove 1 chip on a successful attack [D(A) = D(B) = D(C) = 1]. L(X) was varied according to a prearranged schedule (see Table 5). The rules of the game were identical to the rules governing the game in experiment 1a with one modification. After all three players had made their attack choices, they were required to turn their sampling urn over so that a ball appeared in the plastic bubble. If a white ball appeared, the attack was successful. If a black ball appeared, the attack was unsuccessful. The ratio of black and white balls in each players' sampling urn was manipulated to follow the schedule of L(X) indicated in Table 5. In addition, the probabilistic nature of the game required that each game be played until one or none of the players survived. This resulted from the fact that when two players had chips remaining it was not possible to predict the winner.

Results. In this experiment, as was the case previously, we examined the attack data from the initial move. Figure 7 indicates the predicted probabilities of each player attacking the stronger of his two opponents as a function of L(X)

Insert Figure 7 about here

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for n reflective cycles.

Figure 8 reports the observed probability of attack as compared to the predicted probabilities of attack as a function of L(X).

Insert Figure 8 about here

The number of reflective cycles was computed as it was in experiments 1a and 1b. The observed and predicted probabilities of each player attacking the stronger

Figure 7.

Predicted probabilities of each player attacking the stronger of his two opponents as a function of L(X) for 'n' reflective cycles.

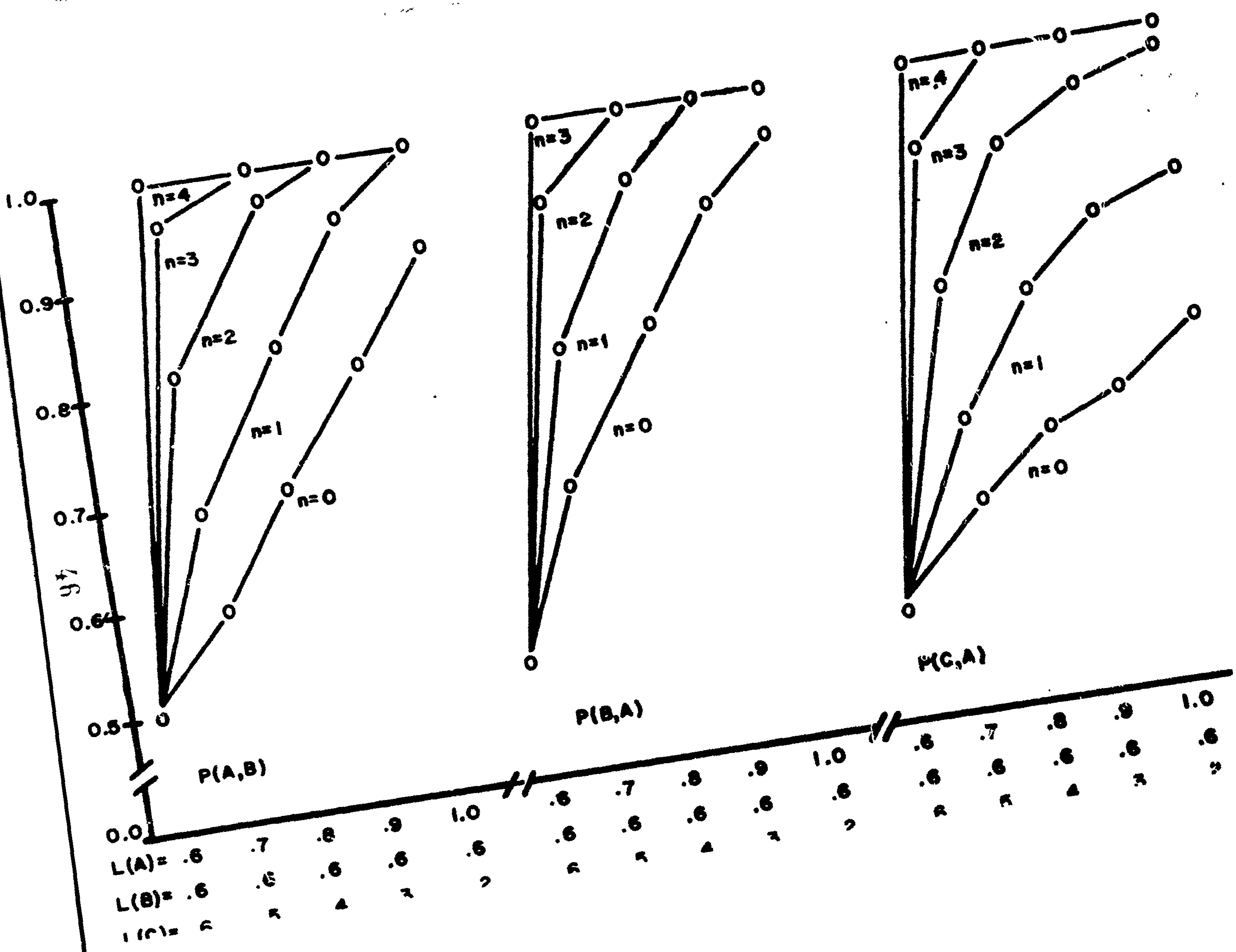
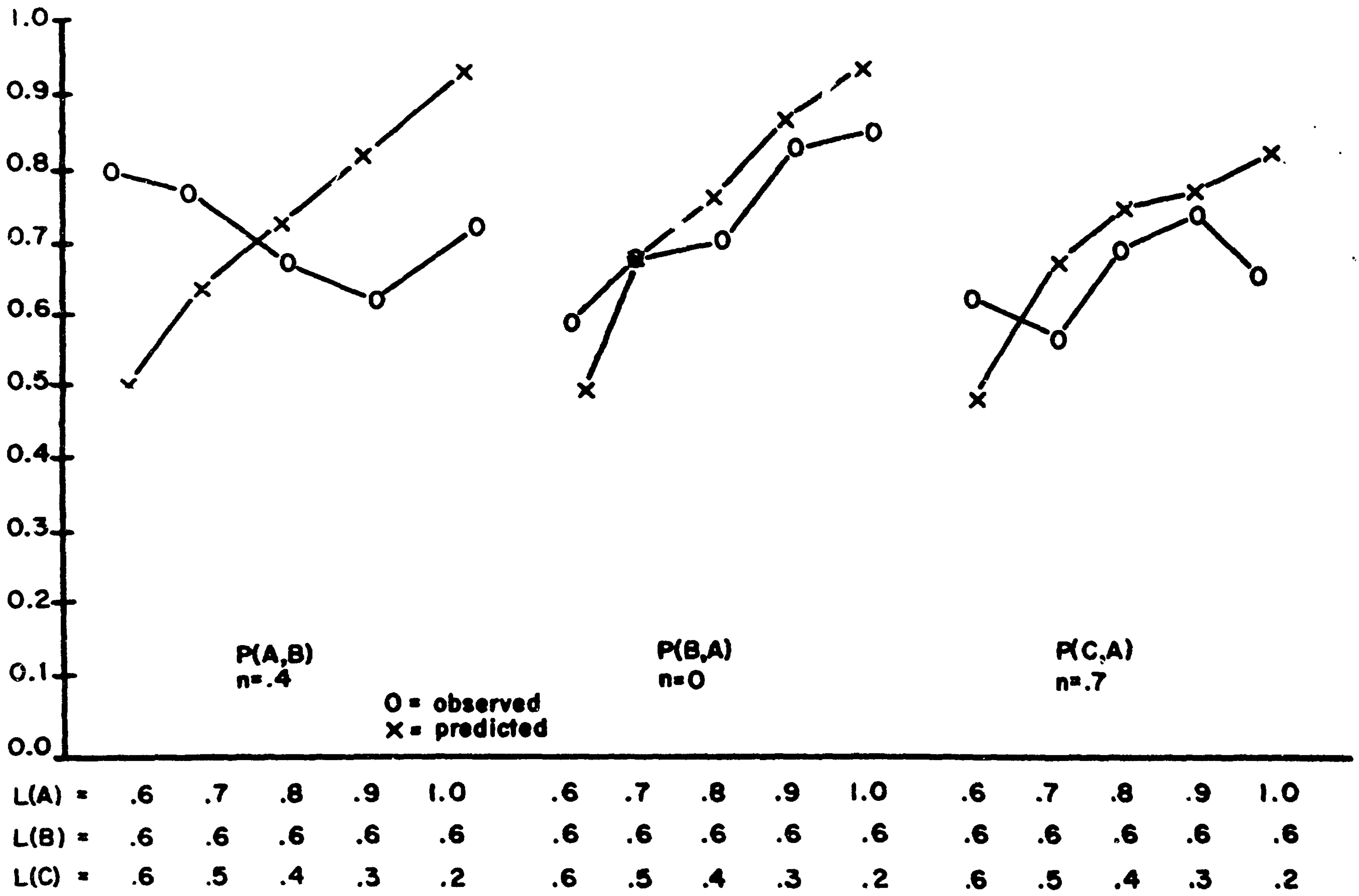

1.0
0.9
0.8
0.7
0.6
0.5
0.0
n=4
n=3
n=2
n=1
n=0
P(A,B)
P(B,A)
P(C,A)
L(A)= .6 .7 .8 .9 1.0 .6 .6 .6 .6 .6
L(B)= .6 .6 .6 .6 .6 .6 .7 .8 .9 1.0
.6 .6 .6 .6 .6 .6 .7 .8 .9 1.0

Figure 8.

Observed and predicted probabilities of each player attacking the stronger of his two opponents as a function of L(X) with 'n' reflective cycles.


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
P(A,B)
n=.4
P(B,A)
n=0
P(C,A)
n=.7
O = observed
X = predicted
L(A) = .6 .7 .8 .9 1.0 .6 .7 .8 .9 1.0 .6 .7 .8 .9 1.0
L(B) = .6 .6 .6 .6 .6 .6 .6 .6 .6 .6 .6 .6 .6 .6 .6
L(C) = .6 .5 .4 .3 .2 .6 .5 .4 .3 .2 .6 .5 .4 .3 .2

of his two opponents as a function of L(X) and n reflective cycles for each move of game types .6/.6/.6, .7/.6/.5, .8/.6/.5, .9/.6/.3, and 1.0/.6/.2 respectively are presented in Figure 9. The number of reflective cycles was computed as it was in experiments 1a and 1b. Any move for which $N < 20$

Insert Figure 9 about here

was not included. Table 6 reports the product-moment correlations between the predicted and observed probabilities.

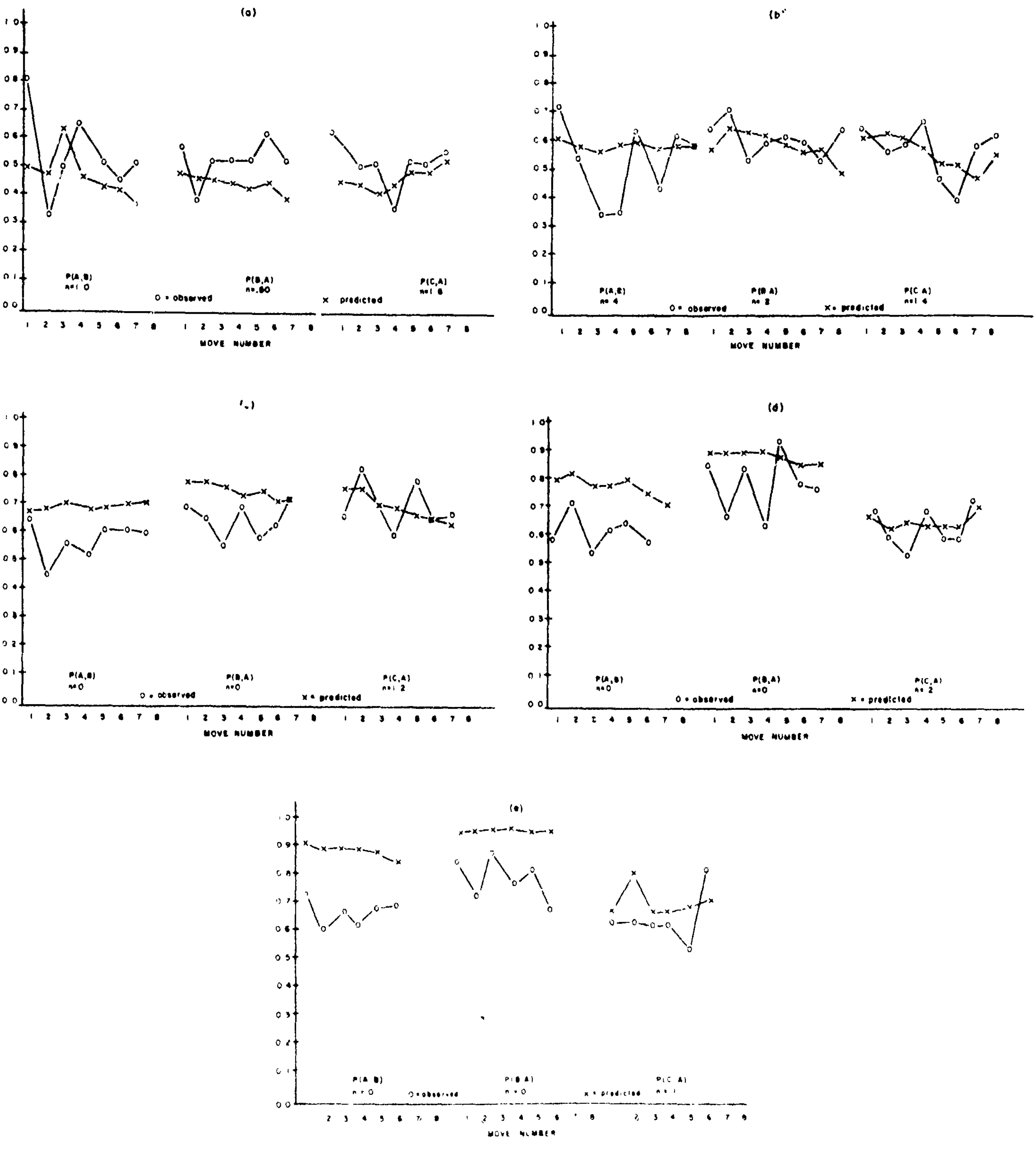
Insert Table 6 about here

Discussion of Experiment 1

The correspondence between the theoretical curves and the observed data from Experiment 1 leaves a great deal to be desired. There are, however, a number of factors which should be noted in evaluating the fit of the model to the data. The model makes certain assumptions about the motives of the subjects, that is, that each subject seeks to be the sole survivor and that, failing to survive, each subject is indifferent to all possible outcomes. This assumption implies that

Figure 9.

Observed and predicted probabilities of each player attacking the stronger of his two opponents as a function of move number and 'n' reflective cycles with the .6/.6/.6 [L(X)] game type appearing in "a", .7/.6/.5 in "b", .8/.6/.4 in "c", .9/.6/.3 in "d", and 1.0/.6/.2 in "e".



(a)
P(A,B)
P(B,A)
P(C,A)
O = observed
X predicted
MOVE NUMBER
(d)
O = observed
X = predicted
MOVE NUMBER
(e)

TABLE 6

Product-Moment Correlations between Predicted and Observed Probabilities of Each Player Attacking the Stronger of his Two Opponents for Each Game Type in Experiment 1c

Player	Game Type				
	1	2	3	4	5
A	.43	.26	.42	-.43	-.14
B	.01	.20	.07	.09	.60
C	.40	.42	.51	.28	.34

the subjects have an appropriate understanding of the game. Moreover, the model ignores certain variables such as retaliation and the attribution of personal characteristics. These assumptions may be considered more as boundry conditions of the model than as substantive assumptions to be tested. Thus, it is important to consider the degree to which these conditions were met in Experiment 1 and how any failure to meet these conditions should effect the evaluation of the model.

On the whole, subjects did appear highly motivated to win the games. This is consistent both with the manner in which they played and with their post-experiment comments, despite the fact that no monetary incentive was provided. However, a number of subjects indicated a preference for being the second player eliminated, i.e., for achieving second place, despite the fact that a pre-condition of uelative conflict is that losing is losing. There is, moreover, some evidence to indicate that subjects did not fully understand the game. For example, consider figure 2. In the 11/6/1 game, the strongest player, (Player A) is assured of winning if he attacks the intermediate player (Playwr B) every time. As is apparent in figure 2, A attacked B less than 90% of the time in this situation. Moreover, in a study conducted subsequent to the present one, Hartman (1970) demonstrated a statistically significant change in the probability of attacking the stronger player after the first few games. Since subjects in Experiment 1 played only a few games, it may be that a substantial part of the data reflects relatively uninformed decisions. Further, this experiment was conducted in a face-to-face situation which would allow subjects to realiate against a given player in a given game for some action taken by that player in a prior game. The model clearly does not take such a contingency into consideration.

Given the above remarks, the relatively poor fit of the model to the data from Experiment 1 can be more appropriately interpreted. Consider first the initial trial data presented in figures 2, 5, and 8. In figure 2, two of the three graphs show an excellent rank-order correlation between observed and predicted data points. In figure 5, two of the three graphs show acceptable fit and the same is true in figure 8. Thus, given the possible sources of departure from boundry conditions, the model appears to provide a reasonable and encouraging fit to the initial trial data.

To the extent that the experimental conditions in Experiment 1 may have diverged from the boundry conditions of the model, this divergence might be expected to effect the latter moves in a game more seriously than the initial trial. This is the case because of the increased likelihood of retaliation effects and because whatever temptation there may have been to play for "second place" would have been more pronounced on later moves.

Reference to figure 3 indicates a surprisingly good correspondence between predicted and observed values over trials for the games in Experiment 1a. Of the 15 product-moment correlations between predicted and observed values in Table 2, 10 are greater than or equal to .65, and 6 are greater than .80. Four of the five lowest correlations occurred in the two games with the greatest disparity of resources. In these games, subjects tended to under-attack their stronger attack choice. This finding is consistent with the previous comments about lack of understanding of the game.

A similar pattern emerge in figure 6, although the correspondence between predicted and observed values was considerably poorer in Experiment 1b. Of the 21 product-moment correlations (Table 4), only five are larger than .65. The five lowest correlations occurred in the three games with the greatest disparity of resources; and particularly with respect to Player A--the strongest member of the triad--there was a marked tendency to under-attack the stronger attack choice. As in Experiment la, this could be accounted for by an insufficient understanding of valid winning strategies.

The fit of the model to over-trials data was poorest in Experiment lc as is indicated by figure 9 and Table 6. All of the 15 correlations were less than .65. The post-experimental comments of the subjects in this experiment indicated that they had failed to appreciate the nature of the probabilistic resource dimension (launch capability) that was varied in Experiment lc.

On the whole, the results reported above, while in no way constituting adequate support for the model, seem to be influenced by a failure to meet the boundry conditions of the model in the experimental conditions. It is recognized that methodological apologies do not provide a very satisfactory substitute for a demonstrated correspondence between theory and data. It was with this in mind that Experiment 2 was designed.

Experiment 2

Subjects. The subjects were four male and two female undergraduates who were paid $1.50 per hour plus bonuses to participate in the experiment.

Apparatus. A talbe divider was used to control the non-face-to-face aspect of the game. It was designed to divide a 2 1/3' X 5' table into four sections so that the subjects could not see each other or the experimenter. One 1 X 4 inch slot in the bottom of each of the dividers between the subjects and the experimenter was provided for written communication.

A 12" X 36" scoreboard was mounted on the wall behind the experimenter in full view of the subjects. Each subject's score was kept by sliding billiard markers on 1/8 rods behind a 18 X 12" cover such that the markers that remained visible indicated the subjects' score at any given point in the game. For ease in calculating the score the markers were placed in a sequence of four white and one black marker. There were a total of thirty markers for each subject.

Procedure. Four male and two female undergraduates were hired to participate in the experiment. Two triads each composed of one female and two males were formed. Each triad was composed of the same players for the duration of the experiment which consisted of five three hour game playing sessions over a three week period. The games were played in a non-face-to-face situation with no communication allowed between subjects.

The truel paradigm described in experiment 1 was used in the present experiment. In every game R(X) was thirty points, I(X) was 0.0, and L(X) was 1.0 for all three players. The six different distributions of D(X) reported in Table 4 were examined. R(X) and the six distributions of D(X) were chosen to regulate the length of the game and to allow an evaluation of the model in

Insert Table 7 about here

situations ranging from an all equal situation -- the 6/6/6 game type -- to a situation in which one player had dictatorial power -- the 1/6/1 game type. Each subject was assigned a "name" or label in each set. In addition, the labels VAF, ZEJ, AND YOV were counterbalanced on the scoreboard such that each label was associated with the top, middle, or bottom row of points an equal number of times. The subjects were assigned to positions around the game divider for each set of thirty-six games. Each subject occupied each of the three positions an equal number of times.

On each move of the game, each player circled the label of the player he wished to attack on an attack choice slip and passed it to the experimenter. After the experimenter had received attack choices from all three players on each move, the players were told who had attacked whom and R(X) was appropriately adjusted for each player. Thus, the moves in the game were simultaneous.

TABLE 7

D(A), D(B), and D(C) and the Number of Games

Played for the Six Game Types in Experiment 2

	Game Type					
	1	2	3	4	5	6
D(A)	6	7	8	9	10	11
D(B)	6	6	6	6	6	6
D(C)	6	5	4	3	2	1
# of Games Played	110	86	84	83	48	47

The winner of each game was that player who had points remaining when the other p;ayers' points were gone, that is, he was the sole survivor. If there was no sole survivor there was no winner.

One point was given to the winning player in each game. After two sessions (approximately 80 games) the number of points accumulated by each player was totaled and in one triad the player with the most points was given a twelve dollar bonus in addition to his hourly wage. The other two members of the triad received only their hourly wage. In the other triad, the players divided a nine dollar bonus in direct proportion to the ratio of the number of points they had accumulated to the total number of points accumulated.

Results. Since there were no differences as a function of the way the bonuses were determined and there were no individual or sex differences, the data were collapsed and examined as a function of D(X) and game type.

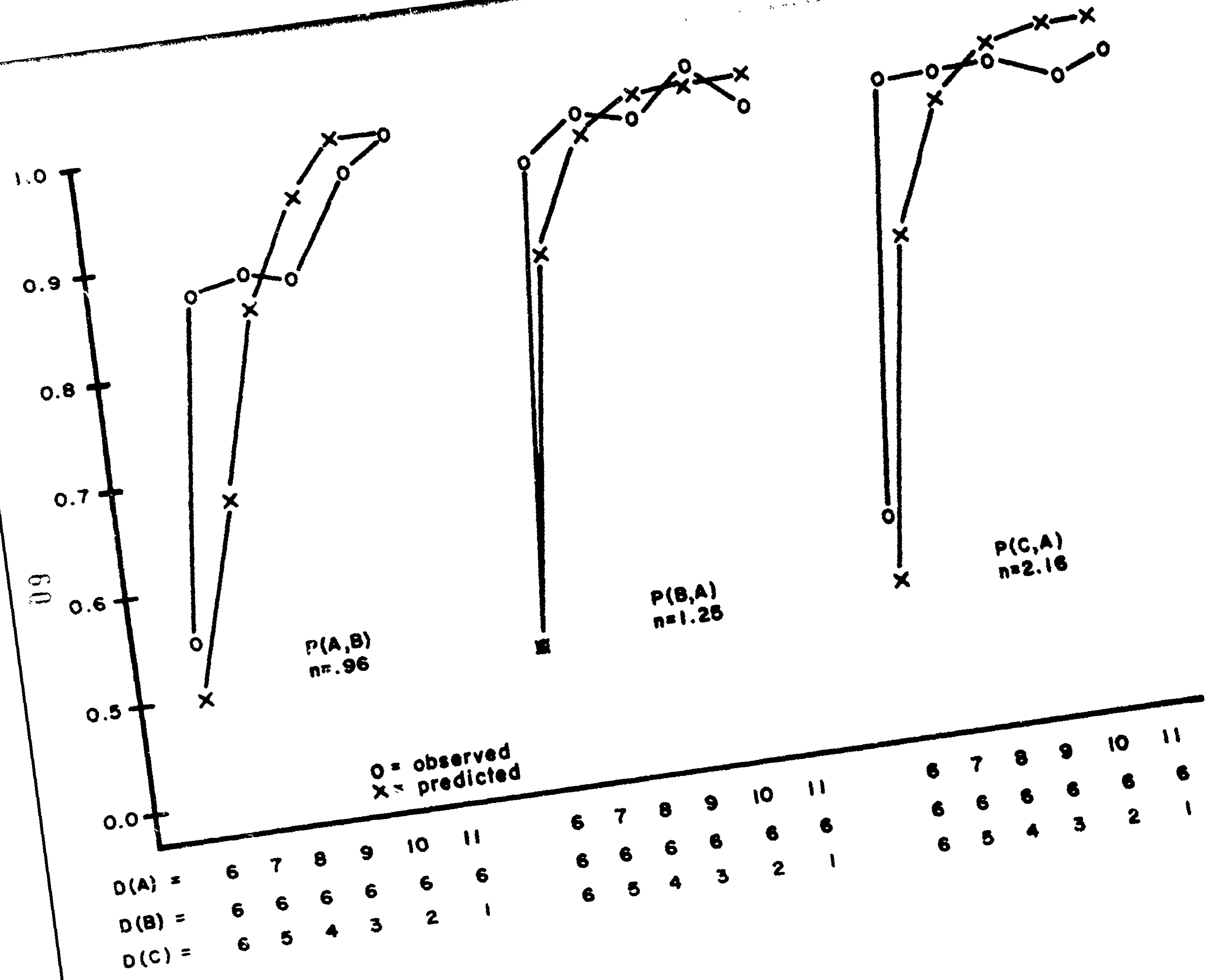
Figure 10 presents this comparison for each of the three power positions in Experiment 2. The three values of n were estimated by solving for n for each power position in each game type and by averaging over game types to obtain a value of n for each power position. In this procedure, the 6/6/6 data was not included because the predicted probability of attack does not change as a function of number of reflective cycles. The 10/6/2 data was also omitted because the number of reflective cycles associated with P(B,A) for this game was undefined.

Insert Figure 10 about here

Figure 11 presents a comparison of the prediction of the model and observed over-trials data. Of the 18 product-moment correlations presented in Table 8, 12 were higher than .65 and 10 were higher than .80.

Figure 10.

Observed and predicted probabilities of each player attacking the stronger of his two opponents as a function of D(X) and 'n' reflective cycles.


1.0
0.9
0.8
0.7
0.6
0.5
0.0
P(A,B)
n=.96
P(B,A)
n=1.25
P(C,A)
n=2.16
O = observed
X = predicted
D(A) = 6 7 8 9 10 11 6 7 8 9 10 11 6 7 8 9 10 11
D(B) = 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6
D(C) = 6 5 4 3 2 1 6 5 4 3 2 1 6 5 4 3 2 1

Insert Figure 11 about here

Insert Table 8 about here

A 3 x 4 factorial analysis of variance with repeated measures on one factor and with one observation per cell was computed on the data in Table 9. Table 10 reports the summary of that analysis. There was a significant tendency for the number of reflective cycles to decrease as the disparity of D(X) between player positions increased ($F = 42.50$, $p < .001$). In addition, a significant propensity for each player position (A, B, and C) to consider a differential number

Insert Table 9 about here

Insert Table 10 about here

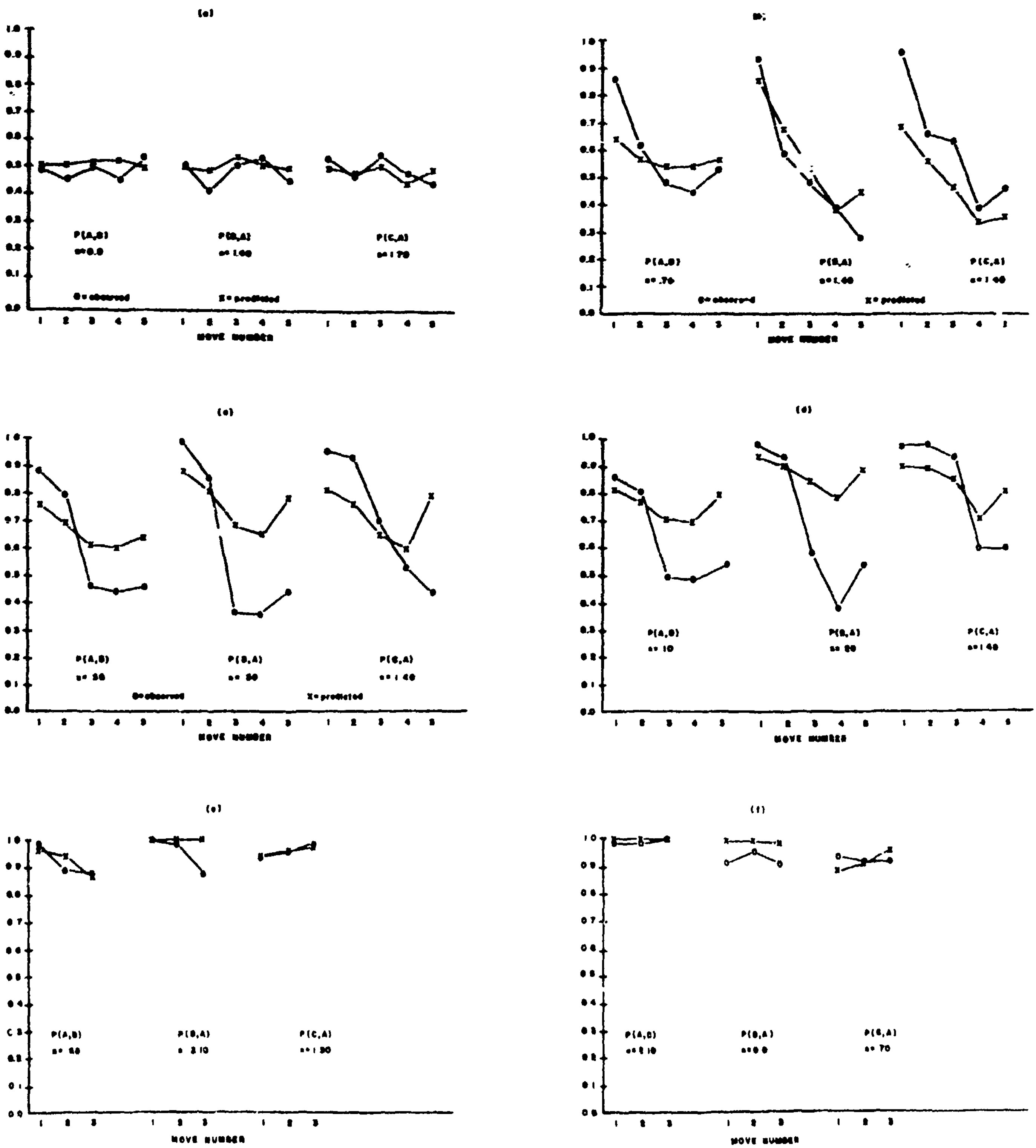
of reflective cycles ($F - 34.83$, $p < .001$) was indicated. An examination of Figure 11 indicates that the difference associated with player position results from a tendency for Player C to consider more reflective cycles than both Players A and B. There is no difference between Players A and B.

Discussion of Experiment 2

The results of Experiment 2 provide at least a partial validation of the methodological crticisms of Experiment 1. When care was taken (a) to provide subjects with sufficient experience to fully understand the game, (b), to provide the monetary incentive to establish the equivalence of all losing outcomes,

Figure 11.

Observed and predicted probabilities of each player attacking the stronger of his two opponents as a function of move number and 'n' reflective cycles with the 6/6/6 [R(X)] game type appearing in "a", 7/6/5 in "b", 8/6/4 in "c", 9/6/3 in "d", 10/6/2 in "e", and 11/6/1 in "f".



(a)
(b)
(c)
(d)
(e)
(f)
P(A,B)
P(B,A)
P(C,A)
MOVE NUMBER

TABLE 8

Product-Moment Correlations between Predicted and Observed Probabilities of Each Player Attacking the Stronger of his Two Opponents for Each Game Type in Experiment 2

Player	Game Type					
	1	2	3	4	5	6
A	-.49	.99	.96	.77	.83	-.99
B	.67	.94	.89	.88	.81	.18
C	.43	.98	.42	.88	1.00	-.80

TABLE 9

Number of Reflective Cycles Across Player Position and Game Type

D(A)	D(B)	D(C)	Player A	Player B	Player C
7	6	5	2.4	2.0	3.4
8	6	4	1.4	1.7	2.5
9	6	3	0.4	0.8	2.2
11	6	1	0.1	0.0	1.4

TABLE 10

Summary Table for the 3 x 4 Factorial Analysis of Variance[1]

Source	SS	df	MS	F
Game Type	7.64	3	2.55	42.50*
Player Position	4.18	2	2.09	34.83*
Error	.34	6	.06	
Total	12.16	11		

[1]Note.--Since the analysis of variance had only one observation per cell the interaction term was used as the error term.

*$p < .001$

and (c) to eliminate cues of personal characteristics and information allowing for inter-game retaliation, the correspondence between theoretical and observed values was greatly enhanced. Figure 10 shows an excellent fit between predicted and observed initial attack probabilities.

Despite the very encouraging results of Experiment 2, we would be mistaken at this point to make anything but very modest claims for the validity or heuristic value of the model. What has emerged from the empirical work of this paper is a conclusion that further tests of the model are warranted.

Uelative conflict as we have characterized it in this paper, stands in the shadows of virtually all other conflict. While it emerges only infrequently or in a limited form, its existence influences attempts at the resolution of conflict. Thus, a better understanding of the nature of uelative conflict would seem to be essential to the management of conflict in less intense situations. This paper is intended to be a step in the direction of that understanding.

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